OCR
Oxford Cambridge and RSA

Accredited

A Level Mathematics A
H240/01 Pure Mathematics
Sample Question Paper

Date – Morning/Afternoon
Time allowed: 2 hours

OCR supplied materials:
- Printed Answer Booklet

You must have:
- Printed Answer Booklet
- Scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.**
- Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r|<1$$

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1\, a^{n-1}b + {}^n\mathrm{C}_2\, a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r\, a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad \left(|x|<1,\ n \in \mathbb{R}\right)$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h\to 0}\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta, \cos\theta \approx 1-\tfrac{1}{2}\theta^2, \tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n,\ p)$ then $P(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, Mean of X is np, Variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.575	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \tfrac{1}{2}at^2$

$s = \tfrac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \tfrac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \tfrac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \tfrac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \tfrac{1}{2}\mathbf{a}t^2$

Answer **all** the questions

1 Solve the simultaneous equations.

$$x^2 + 8x + y^2 = 84$$
$$x - y = 10$$

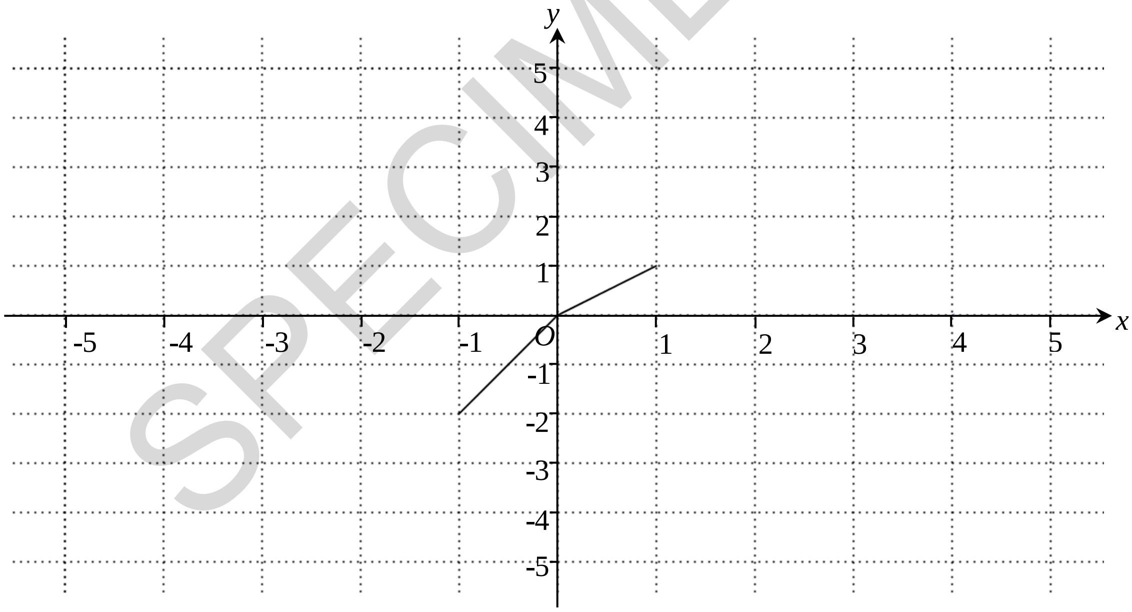
[4]

2 The points A, B and C have position vectors $3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$, $-\mathbf{i} + 6\mathbf{k}$ and $7\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$ respectively. M is the midpoint of BC.

(i) Show that the magnitude of $\overrightarrow{OM}$ is equal to $\sqrt{17}$. **[2]**

(ii) Point D is such that $\overrightarrow{BC} = \overrightarrow{AD}$. Show that position vector of the point D is $11\mathbf{i} - 8\mathbf{j} - 6\mathbf{k}$. **[3]**

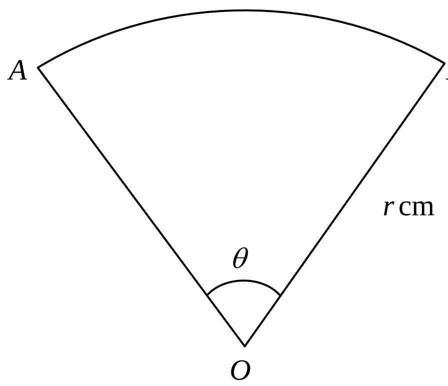
3 The diagram below shows the graph of $y = \mathrm{f}\left(x\right)$.



(i) On the diagram in your Printed Answer Booklet, draw the graph of $y = \mathrm{f}\left(\frac{1}{2}x\right)$. **[1]**

(ii) On the diagram in your Printed Answer Booklet, draw the graph of $y = \mathrm{f}(x-2)+1$. **[2]**

4



The diagram shows a sector AOB of a circle with centre O and radius r cm. The angle AOB is θ radians. The arc length AB is 15 cm and the area of the sector is 45 cm^2.

(i) Find the values of r and θ. **[4]**

(ii) Find the area of the segment bounded by the arc AB and the chord AB. **[3]**

5 **In this question you must show detailed reasoning.**

Use logarithms to solve the equation

$$3^{2x+1} = 4^{100},$$

giving your answer correct to 3 significant figures. **[4]**

6 Prove by contradiction that there is no greatest even positive integer. **[3]**

7 Firm A made a £5000 profit during its first year. In each subsequent year, the profit increased by £1500 so that the profit was £6500 during the second year, £8000 during the third year and so on.

Firm B made a £5000 profit during its first year. In each subsequent year, the profit was 90% of the previous year's profit.

(i) Find an expression, in its simplest form, for the total profit made by firm A during the first n years. **[2]**

(ii) Find an expression, in its simplest form, for the total profit made by firm B during the first n years. **[3]**

(iii) Find how many years it will take for the total profit of firm A to reach £385 000. **[3]**

(iv) Comment on the profits made by each firm in the long term. **[2]**

8 **(i)** Show that $\frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$. **[3]**

(ii) **In this question you must show detailed reasoning.**

Solve $\frac{2\tan\theta}{1+\tan^2\theta} = 3\cos 2\theta$ for $0 \le \theta \le \pi$. **[3]**

9 The equation $x^3 - x^2 - 5x + 10 = 0$ has exactly one real root α.

(i) Show that the Newton-Raphson iterative formula for finding this root can be written as

$$x_{n+1} = \frac{2x_n^{\ 3} - x_n^{\ 2} - 10}{3x_n^{\ 2} - 2x_n - 5}.$$

[3]

(ii) Apply the iterative formula in part **(i)** with initial value $x_1 = -3$ to find x_2, x_3, x_4 correct to 4 significant figures. **[1]**

(iii) Use a change of sign method to show that $\alpha = -2.533$ is correct to 4 significant figures. **[3]**

(iv) Explain why the Newton-Raphson method with initial value $x_1 = -1$ would not converge to α. **[2]**

10 A curve has equation $x = (y+5)\ln(2y-7)$.

(i) Find $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y. **[3]**

(ii) Find the gradient of the curve where it crosses the y-axis. **[5]**

11 For all real values of x, the functions f and g are defined by $\mathrm{f}(x) = x^2 + 8ax + 4a^2$ and $\mathrm{g}(x) = 6x - 2a$, where a is a positive constant.

(i) Find $\mathrm{fg}(x)$. Determine the range of $\mathrm{fg}(x)$ in terms of a. **[4]**

(ii) If $\mathrm{fg}(2) = 144$, find the value of a. **[3]**

(iii) Determine whether the function fg has an inverse. **[2]**

12 The parametric equations of a curve are given by $x = 2\cos\theta$ and $y = 3\sin\theta$ for $0 \le \theta < 2\pi$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ. **[2]**

The tangents to the curve at the points P and Q pass through the point (2, 6).

(ii) Show that the values of θ at the points P and Q satisfy the equation $2\sin\theta + \cos\theta = 1$. **[4]**

(iii) Find the values of θ at the points P and Q. **[5]**

13 **In this question you must show detailed reasoning.**

Find the exact values of the x-coordinates of the stationary points of the curve $x^3 + y^3 = 3xy + 35$. **[9]**

14 John wants to encourage more birds to come into the park near his house. Each day, starting on day 1, he puts bird food out and then observes the birds for one hour. He records the maximum number of birds that he observes at any given moment in the park each day. He believes that his observations may be modelled by the differential equation

$$\frac{\mathrm{d}n}{\mathrm{d}t} = 0.1n\left(1 - \frac{n}{50}\right)$$

where n is the maximum number of birds that he observed at any given moment on day t.

(i) Show that the general solution to the differential equation can be written in the form $n = \frac{50A}{\mathrm{e}^{-0.1t} + A}$, where A is an arbitrary positive constant. **[9]**

(ii) Using his model, determine the maximum number of birds that John would expect to observe at any given moment in the long term. **[1]**

(iii) Write down one possible refinement of this model. **[1]**

(iv) Write down one way in which John's model is not appropriate. **[1]**

END OF QUESTION PAPER

…day June 20XX – Morning/Afternoon

A Level Mathematics A
H240/01 Pure Mathematics

SAMPLE MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 100

This document consists of 16 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. ‘Fresh starts’ will not affect an earlier decision about a misread. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1			$x^2+8x+(x-10)^2=84$ $2x^2-12x+16=0$ $x=2$, $x=4$ $x=2$ and $y=-8$ $x=4$ and $y=-6$	**M1** **A1** **A1** **A1** **[4]**	**1.1a** **1.1b** **1.1** **1.1**	Substitute the linear equation into the quadratic Correctly simplified answer BC, but allow by any valid method Values should be paired correctly	**OR M1** $(y+10)^2+8(y+10)+y^2=84$ **A1** $2y^2+28y+96=0$ **A1** $y=-8$, $y=-6$
2	(i)		$\overrightarrow{OM}=\frac{1}{2}\left(\overrightarrow{OC}+\overrightarrow{OB}\right)=3\mathbf{i}-2\mathbf{j}+2\mathbf{k}$ $\left\lvert\overrightarrow{OM}\right\rvert=\sqrt{3^2+(-2)^2+2^2}=\sqrt{9+4+4}=\sqrt{17}$	**M1** **E1** **[2]**	**1.1** **2.1**	Attempt to find $\overrightarrow{OM}$ AG	
2	(ii)		$\overrightarrow{BC}=8\mathbf{i}-4\mathbf{j}-8\mathbf{k}$ $\overrightarrow{OD}=\overrightarrow{OA}+\overrightarrow{AD}=\overrightarrow{OA}+\overrightarrow{BC}$ $\overrightarrow{OD}=3\mathbf{i}-4\mathbf{j}+2\mathbf{k}+8\mathbf{i}-4\mathbf{j}-8\mathbf{k}$ $=11\mathbf{i}-8\mathbf{j}-6\mathbf{k}$	**M1** **E1** **E1** **[3]**	**1.1** **2.4** **2.1**	 Express $\overrightarrow{OD}$ in terms of known vectors AG An intermediate step must be seen	
3	(i)		Coordinates of vertices seen at (0, 0), (−2, −2) and (2, 1)	**B1** **[1]**	**1.1**	Vertices must be clearly shown	
3	(ii)		Coordinates of vertices seen at (2, 1), (3, 2) and (1, −1)	**M1** **A1** **[2]**	**1.1** **1.1**	Clear attempt to translate graph to the right and to translate it vertically upwards All vertices correct	

Question			Answer	Marks	AO	Guidance	
4	(i)		$r\theta = 15$	B1	1.1		
			$\frac{1}{2}r^2\theta = 45$	B1	1.1		
			$\frac{1}{2}r(15) = 45$	M1	3.1a	Accept any method for solving the equations simultaneously	
			$r = 6$ and $\theta = 2.5$	A1	1.1		
				[4]			
4	(ii)		$\frac{1}{2}(6)^2 \sin\left(\frac{5}{2}\right)$	B1FT	1.1	FT their r and θ	
			$45 - \text{their } \frac{1}{2}(6)^2 \sin\left(\frac{5}{2}\right)$	M1	1.1		
			34.2 (cm^2)	A1FT	1.1	FT their r and θ	
				[3]			
5			**DR**				
			$\log 3^{2x+1} = \log 4^{100}$	*M1	1.1a	Correctly introduce logs (can use any base, if consistent)	**OR**
							M1 $\log_3 3^{2x+1} = \log_3 4^{100}$
			$(2x+1)\log 3 = \log 4^{100}$	A1	1.1	Obtain linear equation in x, with logarithm(s)	**A1** $2x+1 = \log_3 4^{100}$
						allow $2x+1\log 3 = \log 4^{100}$	
			$2x+1 = 126(.18...)$	dep*M1	1.1		
			$x = 62.6$	A1	1.1	cao	
				[4]			
6			Assume that there is a greatest even positive integer $N = 2k$	*E1	2.1	Proof must start with an assumption for contradiction	
			$N + 2 = 2k + 2 = 2(k+1)$	M1	2.1		
			Which is even and $N + 2 > N$ This contradicts the assumption Therefore there can be no greatest even positive integer	dep*E1	2.4	There must be a statement denying the assumption for the final **E1**	
				[3]			

Question			Answer	Marks	AO	Guidance	
7	(i)		Identify AP with $a = 5000$ and $d = 1500$ $\frac{n}{2}(2(5000)+(n-1)1500)$ $= n(750n+4250)$	M1 A1 **[2]**	3.1b 1.1	Identification recognised by an attempt at the sum formula or *n*th term formula for an AP Or $750n^2 + 4250n$	
7	(ii)		$\frac{5000(1-(0.9)^n)}{1-0.9}$ Obtain $50000(1-(0.9)^n)$	M1 A1 A1 **[3]**	3.1b 3.1b 1.1	Identification recognised by an attempt at the sum formula with *n*, $n-1$ or $n+1$ or with a positive sign in numerator Obtain correct unsimplified sum Or $50000 - 50000(0.9)^n$	
7	(iii)		Obtain $750n^2 + 4250n - 385000 = 0$ $n = 20$ or $n = -\frac{77}{3}$ State 20 years	M1 A1 A1 **[3]**	3.1b 1.1 3.4	Equate to 385 000 and solve a 3 term quadratic $= 0$ BC both required Allow different methods for solving the quadratic	**OR** **M1** For writing down and summing the total profit for at least the first four years (may be implied BC) **A1** For finding that the total is equal to 385 000 for *n* = 20 **A1** state 20 years
7	(iv)		Firm A’s profits continue to grow Firm B’s profits eventually plateau at £50 000 as $(0.9)^n$ tends to 0 with large enough *n*	E1 E1 **[2]**	3.4 3.2a	 Some mention is required about the effect of $(0.9)^n$	

Question			Answer	Marks	AO	Guidance	
8	(i)		$\frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\sin\theta}{\cos\theta} \div \sec^2\theta$	B1	2.1	Use $1+\tan^2\theta = \sec^2\theta$ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$	
			$= \frac{2\sin\theta\cos^2\theta}{\cos\theta}$	M1	2.1	Express LHS in terms of $\sin\theta$ and $\cos\theta$	**M0** for attempts to rearrange to solve an equation
			$= 2\sin\theta\cos\theta = \sin 2\theta$	A1	2.2a		
				[3]			
8	(ii)		**DR** $\sin 2\theta = 3\cos 2\theta$ so $\tan 2\theta = 3$	B1	2.2a	Use the result of **(i)** or otherwise achieve an equation in tan only	**OR B1** for squaring both sides and achieving an equation in either sin or cos only
			$\theta = \frac{1}{2}\tan^{-1}3$ oe	M1	2.1	Use correct order of operations to solve, must be shown	
			0.625, 2.20	A1	1.1	Both values required. May be given to 3 s.f. or better (0.624523, 2.195319), or both solutions in exact form $\frac{1}{2}\tan^{-1}3$, $\frac{1}{2}\tan^{-1}3+\frac{1}{2}\pi$	For answers alone award no marks
				[3]			

Question			Answer	Marks	AO	Guidance	
9	(i)		$f'(x) = 3x^2 - 2x - 5$	B1	1.1		
			$x_{n+1} = x_n - \frac{x_n^3 - x_n^2 - 5x_n + 10}{3x_n^2 - 2x_n - 5}$	M1	1.1	Substitute into correct formula for Newton-Raphson	
			$x_{n+1} = \frac{3x_n^3 - 2x_n^2 - 5x_n - (x_n^3 - x_n^2 - 5x_n + 10)}{3x_n^2 - 2x_n - 5} =$ $= \frac{2x_n^3 - x_n^2 - 10}{3x_n^2 - 2x_n - 5}$	E1 [3]	2.1	AG a correct intermediate step leading to the given answer is required	
9	(ii)		$x_2 = -2.607$ $x_3 = -2.535$ $x_4 = -2.533$	B1 [1]	1.1	BC All three values must be given to 4 significant figures.	
9	(iii)		$f(-2.5325)$ and $f(-2.5335)$	M1	1.1	Accept other alternative values which would confirm α as a root correct to 4 s.f.	
			$(-2.5325)^3 - (-2.5325)^2 - 5(-2.5325) + 10 =$ 0.0066125 $(-2.5335)^3 - (-2.5335)^2 - 5(-2.5335) + 10 =$ –0.0127017	A1	2.1	At least the result of evaluation must be shown	
			Since $f(-2.5325) > 0$ and $f(-2.5335) < 0$ x_4 is α to 4 s.f.	E1 [3]	2.4	The change of sign must be pointed to	
9	(iv)		$3(-1)^2 - 2(-1) - 5 = 0$	B1	2.1		
			Since the fraction is undefined at $x = -1$, x_2 is undefined	E1 [2]	1.2	Accept references to a stationary point of the function	or the tangent to the curve being horizontal

Question			Answer	Marks	AO	Guidance	
10	(i)		Attempt use of product rule	M1	1.1a	Award for sight of two terms	
			Obtain $\ln(2y-7)$	A1	1.1		
			Obtain.... $+\dfrac{2(y+5)}{2y-7}$	A1	1.1		
				[3]			
10	(ii)		$(y+5)\ln(2y-7)=0$ $y=-5$ or $y=4$	M1	1.1	Substitute $x=0$ and attempt to solve	
			Substitute $y=4$ into $\dfrac{dx}{dy}(=\ln 1+18)$	M1	3.1a	May attempt to form $\dfrac{dy}{dx}$ by attempting to form the reciprocal. Allow any attempt however poor	
			Obtain $\dfrac{dy}{dx}=\dfrac{1}{18}$	A1	1.1		
			Substitute $y=-5$ into $\dfrac{dx}{dy}$ (or x)	M1	2.1	Do not allow $\ln\lvert-17\rvert$	
			and indicate that $\ln(-17)$ does not exist	A1	2.3	May state that the ln graph does not exist for negative values or at (0, −17)	
				[5]			
11	(i)		$\text{fg}(x)=(6x-2a)^2+8a(6x-2a)+4a^2$ $=36x^2+24ax-8a^2$	B1	1.1	Accept unsimplified form	**OR** **M1** Complete a square on $\text{f}(x)$
			$(\text{fg})'(x)=72x+24a=0$	M1	1.1	Differentiate their $\text{fg}(x)=0$ or use square completion: $4(9x^2+6ax-2a^2)=4(3x+a)^2-4a^2$ $-8a^2$	**A1** Obtain $(x+4a)^2-12a^2$

Question			Answer	Marks	AO	Guidance	
			$x=-\frac{a}{3}$, giving $\text{fg}\left(-\frac{a}{3}\right)=(-4a)^2+8a(-4a)+4a^2=-12a^2$	**M1**	**2.1**	Solve for x and substitute their value for x in $\text{fg}(x)$	**M1** Substitute $\text{g}(x)$ and simplify
			Stationary point of fg is a minimum so range of $\text{fg}(x)\geq -12a^2$ or $\left[-12a^2\right]$	**E1** **[4]**	**2.2a**	Must mention minimum Do not accept $x\geq -12a^2$	**E1** Obtain $(6x+2a)^2-12a^2$ or equivalent form and state $\text{fg}(x)\geq -12a^2$
11	**(ii)**		$144+48a-8a^2=144$	**M1**	**3.1a**	Substitute $x=2$ in their $\text{fg}(x)$ and equate to 144	
				M1	**1.1**	Attempt to solve their equation	
			$a=6$	**A1** **[3]**	**1.1**	Do not give this mark if $a=0$ also given as an answer	
11	**(iii)**		Each y value in the range ($y>-12a^2$) corresponds to two x values, e.g. $y=0$ corresponds to $x=1.46$ or -5.46	**M1**	**2.4**	An example or graph must be given, or a clear explanation that quadratic functions on the real numbers are one-to-many.	
			Therefore fg has no inverse	**E1** **[2]**	**2.2a**		

Question			Answer	Marks	AO	Guidance	
12	(i)		Use $\frac{dy}{dx}=\frac{dy}{d\theta}\div\frac{d\theta}{dx}$	M1	1.1a		
			Obtain $\frac{-3\cos\theta}{2\sin\theta}$	A1	1.1		
				[2]			
12	(ii)		$(y-3\sin\theta)=\frac{-3\cos\theta}{2\sin\theta}(x-2\cos\theta)$	M1	3.1a	Attempt equation of straight line in any unsimplified form Accept *x*, *y* confusion	**OR** **M1** When $\theta=\theta_Q$, gradient of curve is given by $\frac{-3\cos\theta_Q}{2\sin\theta_Q}$
			$2y\sin\theta-6\sin^2\theta=-3x\cos\theta+6\cos^2\theta$	M1	1.1	Simplify their equation and use $\cos^2\theta+\sin^2\theta=1$	**M1** The gradient of the line through $(2,6)$ and $(2\cos\theta_Q,3\sin\theta_Q)$ is $\frac{3\sin\theta_Q-6}{2\cos\theta_Q-2}$
			$2y\sin\theta+3x\cos\theta=6$	A1FT	1.1		**M1** Equate and clear fractions
			$12\sin\theta+6\cos\theta=6\Rightarrow 2\sin\theta+\cos\theta=1$	E1 [4]	2.1	Substitute (2, 6) and simplify to AG	**E1** Obtain AG
12	(iii)		Use $R\sin(\theta+\alpha)$ on $2\sin\theta+\cos\theta$ $R\sin\alpha=1,\ R\cos\alpha=2$	M1	3.1a	Should go as far as finding R and α Allow alternative forms	**OR** **M1** Square and use $\sin^2\theta+\cos^2\theta=1$
			Obtain $\alpha=0.4636$ and $R=\sqrt{5}$	A1	1.1		**A1** $4\sin^2\theta+4\sin\theta(1-2\sin\theta)$ $+(1-\sin^2\theta)=1$
			Use correct order of operations to solve $\sqrt{5}\sin(\theta+0.4636)=1$	M1	1.1	Attempt to solve their $R\sin(\theta+\alpha)$	**M1** Simplify and solve $5\sin^2\theta-4\sin\theta=0$

Question			Answer	Marks	AO	Guidance	
			Obtain 0 Obtain 2.21	**B1** **A1** **[5]**	**2.2a** **1.1**	 Or better (2.214345...)	
13			**DR**				
			$3x^2+3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$	**B1**	**1.1**	Attempt LHS derivative	Two non-constant terms
			$=3y+3x\frac{\mathrm{d}y}{\mathrm{d}x}$	**M1** **A1**	**3.1a** **1.1**	Attempt product rule on RHS Correct on RHS	
			To find the stationary points let $\frac{\mathrm{d}y}{\mathrm{d}x}=0$	**E1**	**2.1**	Explicitly set their derivative equal to zero	
			$y=x^2$	**M1**	**3.1a**	Attempt to solve for their y or their x	Alternate $x=y^{\frac{1}{2}}$
			$x^3+\left(x^2\right)^3=3x\left(x^2\right)+35$ $x^6-2x^3-35=0$	**M1**	**2.1**	Substitute to get their polynomial in one variable	Alternate $y^3-2y^{\frac{3}{2}}-35=0$
			Let $p=x^3$, then $p^2-2p-35=0$	**M1**	**2.1**	Transform their disguised quadratic	
			$p=7$ or -5	**M1**	**1.1**	Solve their 3 term quadratic	
			$\Rightarrow x=\sqrt[3]{7}$ or $x=-\sqrt[3]{5}$	**A1**	**3.2a**	For both correct	**A0** for decimal answer
				[9]			

Question			Answer	Marks	AO	Guidance	
14	(i)		E.g. $\int \frac{50}{50n-n^2}\,dn = 0.1\int dt$	M1	1.1a	Attempt to separate variables	
				M1	3.1a	Attempt to use partial fractions on LHS	
			$\int\left(\frac{1}{n}+\frac{1}{50-n}\right)dn = 0.1\int dt$	A1	1.1		
			$\ln n - \ln(50-n) = 0.1t + c$	M1	3.1a	Integrate both sides providing LHS contains a ln expression	
			$\ln\frac{n}{50-n} = 0.1t + c$	M1	1.1	Use log law on LHS	
			$\frac{n}{50-n} = Ae^{0.1t}$	M1	3.1a	Apply inverse of ln and deal with $+c$ Accept e^c oe	
				M1	1.1	Make n the subject of their expression	
			$n = \frac{50Ae^{0.1t}}{1+Ae^{0.1t}}$	A1	1.1	Accept e^c oe	
			$n = \frac{50A}{e^{-0.1t}+A}$	E1 [9]	1.1	Multiply numerator and denominator by $e^{-0.1t}$. AG	
14	(ii)		As t becomes large, $e^{-0.1t}$ becomes approximately 0, A cancels and so 50 birds are expected in the long term	E1 [1]	3.4	50 seen www	
14	(iii)		E.g. Only allow integer values of t E.g. Include an initial value for A E.g. John could record the maximum number of each species that he sees.	E1 [1]	3.5c	For one refinement	

Question			Answer	Marks	AO	Guidance	
14	(iv)		E.g. The model is continuous not discrete E.g. It treats all birds of any species as equivalent, but they will respond to the food in different ways.	E1 [1]	3.5a		

Assessment Objectives (AO) Grid

Question	AO1	AO2	AO3(PS)	AO3(M)	Total
1	4				**4**
2(i)	1	1			**2**
2(ii)	1	2			**3**
3(i)	1				**1**
3(ii)	2				**2**
4(i)	3		1		**4**
4(ii)	3				**3**
5	4				**4**
6		3			**3**
7(i)	1		1		**2**
7(ii)	1		2		**3**
7(iii)	1		1	1	**3**
7(iv)			1	1	**2**
8(i)		3			**3**
8(ii)	1	2			**3**
9(i)	2	1			**3**
9(ii)	1				**1**
9(iii)	1	2			**3**
9(iv)	1	1			**2**
10(i)	3				**3**
10(ii)	2	2	1		**5**
11(i)	2	2			**4**
11(ii)	2		1		**3**
11(iii)		2			**2**
12(i)	2				**2**
12(ii)	2	1	1		**4**
12(iii)	3	1	1		**5**
13	3	3	3		**9**
14(i)	6		3		**9**
14(ii)				1	**1**
14(iii)				1	**1**
14(iv)				1	**1**
Totals	**53**	**26**	**16**	**5**	**100**

PS = Problem Solving
M = Modelling